arXiv:1205.5279v3 [hep-th] 9 Jan 2013

Three-point functions of BMN operators at weak and strong coupling II. One loop matching.

Gianluca Grignani,[a] **A. V. Zayakin**[a,b]

[a]*Dipartimento di Fisica, Università di Perugia,*
I.N.F.N. Sezione di Perugia,
Via Pascoli, I-06123 Perugia, Italy

[b]*Institute of Theoretical and Experimental Physics,*
B. Cheremushkinskaya ul. 25, 117259 Moscow, Russia

E-mail: grignani@pg.infn.it, a.zayakin@gmail.com

ABSTRACT: In a previous paper [JHEP 06 (2012) 142] we have shown that the fully dynamical three-point correlation functions of BMN operators are identical at the tree level in the planar limit of perturbative field theory and, on the string theory side, calculated by means of the Dobashi-Yoneya three string vertex in the Penrose limit. Here we present a one-loop calculation of the same quantity both on the field-theory and string-theory side, where a complete identity between the two results is demonstrated.

Contents

1 Introduction

Three-point correlators in gauge/gravity duality have become a very hot topic in the last couple of years. A variety of fundamental results for the three-point functions have been obtained by means of string theory quasiclassics in the strong coupling limit [1–19], exact string theory Hamiltonian in the Penrose limit, Bethe Ansatz at strong and weak coupling [20–26], perturbative large-N field theory [27, 28], and comparison between weakly coupled planar field theory and results in semiclassical string theory [29–32].

Comparing the gravity side and the gauge side directly is possible only under very specific asymptotic conditions such as, *e.g.*, the Frolov–Tseytlin limit [33]. In this limit it is the smallness of $\lambda' \equiv \frac{\lambda}{J^2}$, where λ is the 't Hooft coupling, and J is an R-charge, that permits a comparison. The limit is, in fact, compatible with both small and large λ. Thus, if for example one deals with a small number of impurities, one can calculate the correlator on the field theory side where $\lambda \to 0$, and at the same time use the full string interaction Hamiltonian in the Penrose limit, where the coupling is large, $\lambda \to \infty$, to obtain legitimately comparable results. In this paper we will be looking for the λ' corrections to three-point correlation functions for states with few excitations, both at weak and at strong coupling.

It has been discussed at length in [30] whether the one-loop corrections for three-point functions are expected to match, even at the leading order. The limits $\lambda \gg 1$ and $\lambda \ll 1$ are normally incompatible, so that even the tree-level matching is not necessarily to be expected [8, 21]. In the case of 2-point functions, it is well-known that there is an agreement between the anomalous dimension of certain gauge theory operators and the energy of the dual string states directly computed from the string sigma model. In [34], an argument for the seemingly coincidental matching up to and including the one-loop correction was provided for states in the $SU(2)$ sector of $\mathcal{N} = 4$ SYM. The argument of [34] goes as follows: consider a near-BPS state, with $E - J \ll 1$ and $J \gg 1$. This limit is feasible both on the string side and on the field theory side. On the string theory side quantum corrections to the semiclassical configuration become suppressed and the limit remains valid. However, it was also discussed

in [30] that this argument, being perfectly valid for the two-point functions, does not apply to the three-point functions, thus there will generally be gauge-field terms of order $\frac{\lambda^0}{J^2}$, whereas string theory yields $\frac{\lambda}{J^2}$. Thus one-loop matching is in general not expected and in fact it is not found in a particular example of a heavy-heavy light three-point function [30]. As far as we know, this has been the only example of an explicit one-loop comparison between perturbative field theory and semiclassical string theory so far. The mismatch might however be due to the difficult identification of the two loop corrected gauge theory states.

In [23] a general formula for three point correlators of single trace operators with arbitrary number of impurities N_i, that satisfy $N_1 = N_2 + N_3$, is provided at one-loop. Two operators are long (and highly excited) and the other is shorter. An amazing matching is seen numerically in the limit of $N_3 \to \infty$, where the semiclassical calculation fully conforms to the Bethe Ansatz calculation.

In this paper we perform an explicit one-loop check of the matching in a different sector, where the operator-state identification between gauge and string theory is perfectly well-defined [35]. The main object of our analysis are two-magnon BMN operators

$$\mathcal{O}_{ij,n}^J = \frac{1}{\sqrt{JN^{J+2}}} \sum_{l=0}^J \operatorname{Tr}\left(\phi_i Z^l \phi_j Z^{J-l}\right) \psi_{n,l}, \tag{1.1}$$

which fall into the three irreducible representations of $SO(4)$

$$\mathbf{4} \otimes \mathbf{4} = \mathbf{1} \oplus \mathbf{6} \oplus \mathbf{9}, \tag{1.2}$$

where $\mathbf{1}$ is the trace (T), $\mathbf{6}$ is the antisymmetric (A), $\mathbf{9}$ is the symmetric traceless representation (S). The wave-functions for different representations are

$$\psi_{n,l}^S = \cos \frac{(2l+1)\pi n}{J+1},$$

$$\psi_{n,l}^A = \sin \frac{2(l+1)\pi n}{J+2}, \tag{1.3}$$

$$\psi_{n,l}^T = \cos \frac{(2l+3)\pi n}{J+3}.$$

We consider three operators: $\mathcal{O}_1 = \mathcal{O}_{n_1}^{J_1,12}, \mathcal{O}_2 = \mathcal{O}_{n_2}^{J_2,23}, \mathcal{O} = \mathcal{O}_n^{J,31}$, where n_1, n_2, n_3 are the operator momenta, J_1, J_2, J_3 are their R-charges R_3, $J = J_1 + J_2$, $J_1 = Jy, J_2 = J(1-y)$. The flavor indices are chosen as $(12), (23), (31)$ to represent the symmetric sector of the theory. The symmetric states are more interesting for our analysis since they provide a non-trivial test of the calculation by requiring cancellation of the J^2 and J order of the one-loop correction. Unlike the trace state, the traceless symmetric state is advantageous since one avoids the complications with subtraction of $\operatorname{Tr} Z^{J+1}\bar{Z}$. Our operators are orthonormalized at tree level, therefore the correlator coincides with the structure constant. We shall be looking for the quantity

$$C_{123} = \langle \bar{\mathcal{O}}_3 \mathcal{O}_1 \mathcal{O}_2 \rangle \tag{1.4}$$

as a function of y, J, n_1, n_2, n_3. This gives the three point correlator of these operators, thanks to the conformal invariance of $\mathcal{N} = 4$ SYM. We shall calculate this correlator on the field theory side as well as on the string theory side. In our previous work [26] we have already shown that at tree level these correlators do coincide with the corresponding quantities computed from the string side. Now we proceed to derive the one-loop contributions.

2 String theory calculation

In terms of the BMN basis $\{\alpha_m\}$ the operators in question look like states

$$\mathcal{O}_m = \alpha_m^\dagger \alpha_{-m}^\dagger |0\rangle \tag{2.1}$$

The three-point function is related to the matrix element of the Hamiltonian as follows

$$\langle \bar{\mathcal{O}}_3 \mathcal{O}_1 \mathcal{O}_2 \rangle = \frac{4\pi}{-\Delta_3 + \Delta_1 + \Delta_2} \sqrt{\frac{J_1 J_2}{J}} H_{123} \tag{2.2}$$

where

$$\Delta_1 = J_1 + 2\sqrt{1 + \lambda' n_1^2},$$

$$\Delta_2 = J_2 + 2\sqrt{1 + \lambda' n_1^2}, \tag{2.3}$$

$$\Delta_3 = J + 2\sqrt{1 + \lambda' n_3^2},$$

and the matrix element is defined as

$$H_{123} = \langle 123|V\rangle. \tag{2.4}$$

There has been some ambiguity in the literature with regard to how the proper prefactor in the vertex function V in the pp-wave looks like [36–48]; we use the findings of [48] to start with the Dobashi–Yoneya prefactor [43] in the natural string basis $\{a_m^r\}$.

$$V = P e^{\frac{1}{2} \sum_{m,n} N_{mn}^{rs} \delta^{IJ} a_m^{rI\dagger} a_n^{sJ\dagger}}. \tag{2.5}$$

Here I, J are $SU(4)$ flavour indices, r, s run within $1, 2, 3$ and refer to the first, second and third operator. The natural string basis is related to the BMN basis for $m > 0$ as follows

$$\begin{cases} \alpha_m = \dfrac{a_m + i a_{-m}}{\sqrt{2}} \\[2mm] \alpha_{-m} = \dfrac{a_m - i a_{-m}}{\sqrt{2}} \end{cases}, \tag{2.6}$$

The Neumann matrices are given as [40]

$$N_{m,n}^{rs} = \frac{1}{2\pi} \frac{(-1)^{r(m+1)+s(n+1)}}{x_s\omega_{rm} + x_r\omega_{sn}} \sqrt{\frac{x_r x_s(\omega_{rm} + \mu x_r)(\omega_{sn} + \mu x_s)s_{rm}s_{qn}}{\omega_{rm}\omega_{sn}}},$$

$$N_{-m,-n}^{rs} = -\frac{1}{2\pi} \frac{(-1)^{r(m+1)+s(n+1)}}{x_s\omega_{rm} + x_r\omega_{sn}} \sqrt{\frac{x_r x_s(\omega_{rm} - \mu x_r)(\omega_{sn} - \mu x_s)s_{rm}s_{qn}}{\omega_{rm}\omega_{sn}}}, \tag{2.7}$$

where m, n are always meant positive,

$$\begin{aligned} s_{1m} &= 1, \\ s_{2m} &= 1, \\ s_{3m} &= -2\sin(\pi m y), \end{aligned} \tag{2.8}$$

and

$$\begin{aligned} x_1 &= y, \\ x_2 &= 1 - y, \\ x_3 &= -1, \end{aligned} \tag{2.9}$$

the frequencies of the string oscillators are then given as

$$\omega_{r,m} = \sqrt{m^2 + \mu^2 x_r^2}, \tag{2.10}$$

and the parameter μ is directly related to the Frolov-Tseytlin expansion parameter λ'

$$\mu = \frac{1}{\sqrt{\lambda'}}. \tag{2.11}$$

The one-loop calculation of the correlation function will amount a next-order expansion in $\frac{1}{\mu^2}$ of the matrix element. An essential feature of the Dobashi-Yoneya prefactor we are using is that the prefactor is supported with positive modes only

$$P = \sum_{m>0} \sum_{r,I} \frac{\omega_r}{\mu\alpha_r} a_m^{Ir\dagger} a_m^{Ir}. \tag{2.12}$$

Due to the flavour structure of C_{123} the only combinations of terms from the exponent that could contribute are $N_{n_1 n_2}^{12} N_{n_2 n_3}^{23} N_{n_3 n_1}^{31}$. The leading order contribution is

$$C_{123}^0 = \frac{1}{\pi^2} \frac{\sqrt{J}}{N} \frac{n_3^2 y^{3/2}(1-y)^{3/2}\sin^2(\pi n_3 y)}{(n_3^2 y^2 - n_1^2)(n_3^2(1-y)^2 - n_2^2)} \tag{2.13}$$

The overall factor -4 difference with [26] is due to wave-function normalization. The next-order coefficient in the expansion

$$C_{123} = C_{123}^0 \left(1 + \lambda' c_{123}^1\right), \tag{2.14}$$

where $c_{123}^1 \equiv \frac{C_{123}^1}{C_{123}^0}$ is

$$c_{123}^1 = -\frac{1}{4}\left(\frac{n_1^2}{y^2} + \frac{n_2^2}{(1-y)^2} + n_3^2\right). \tag{2.15}$$

Let us compare this calculation to the field theory calculation.

3 Planar Field Theory at One Loop

To calculate the correlation function at one loop level we use the technique developed in [49, 50]. The leading order correlation function was calculated by us in our preceding paper [26] and is given by the diagram of Fig. (1).
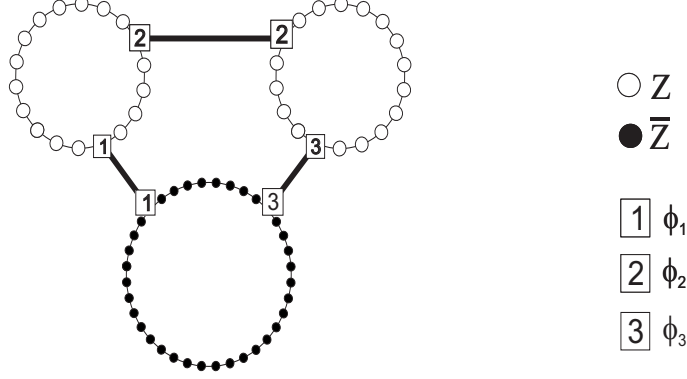

Figure 1. Leading order diagram for the three-point correlation function of the fully dynamic BMN operators from the symmetric traceless sector.

Only ϕ propagators are shown explicitly in the figure; planarity is imposed, the Z fields do not have any extra choice than to contract a definite $\bar{Z}$, thus the diagram contributes exactly once. The diagram (1) refers to the correlation function calculated from planar perturbation theory and is related to the C_{123} defined above

$$S = N\sqrt{J_1 J_2 J} C_{123} \tag{3.1}$$

which is given in general by a λ' expansion

$$S = S^{(0)} + S^{(1)} + O(\lambda'^2) \tag{3.2}$$

and evaluates in the leading order to

$$S^{(0)} = \sum_{l_1=0,l_2=0}^{l_1=J_1,L_2=J_2} \cos\frac{\pi(2l_1+1)}{J_1+1} \cos\frac{\pi(2l_2+1)}{J_2+1} \cos\frac{\pi(2(l_1+l_2)+1)}{J+1}, \tag{3.3}$$

which after the $1/J$ expansion and the due normalization of the operator to unity yields

$$C_{123}^0 = \frac{1}{\pi^2} \frac{\sqrt{J}}{N} \frac{n_3^2 y^{3/2}(1-y)^{3/2}\sin^2(\pi n_3 y)}{(n_3^2 y^2 - n_1^2)(n_3^2(1-y)^2 - n_2^2)}, \tag{3.4}$$

corresponding exactly to the result above.

At the one loop level we estimate the next-order terms in the λ' expansion $S^{(1)}$ considering all possible insertions of the interaction terms

$$H_2 = \frac{\lambda}{8\pi^2}(I - P) \tag{3.5}$$

into Fig. (1), where I is unity operator and P permutation operator, both acting on nearest neighbors. It is convenient to split the Hamiltonian into the part P and the part I; in the diagrams below the four-point vertices are meant as pure permutations, i.e. acting as P solely. The set of the resulting eight diagrams that contribute to C^1_{123} are shown in Fig. (2). Only the insertions and the $\phi^{1,2,3}$ propagators are shown; the configuration of the rest of the propagators is fully determined by planarity rules.

Three types of mixing aggravate our task: the admixture of multi-trace operators (eq. (3.14) in [50]), magnon mode number non-conserving admixture caused by the coupling dependent wave function correction (BMN operator redefinition, eq. (5.18) in [51]), and the admixture with fermionic operators [52, 53],

Multi-trace operator redefinition is organized as

$$\mathcal{O}_n^{J,12\prime} = \mathcal{O}_{12,n}^{J,12} - \frac{J^2}{N} \sum_{k,r} \frac{r^{3/2}\sqrt{1-r}\sin^2(\pi nr)k}{\sqrt{J}\pi^2(k-nr)^2(k+nr)} \mathcal{T}_{12,k}^{J,r} \tag{3.6}$$

where $\mathcal{T}_{12,n}^{J,r} = \mathcal{O}_n^{rJ,12}\mathcal{O}^{(1-r)J}$, $\mathcal{O}^J$ being the normalized vacuum operator of length J. For our kinematics the multi-trace mixing becomes significant only in the next-order corrections in $1/N$.

The magnon mode number nonpreserving BMN operator $\mathcal{O}_n$ redefinition in the order λ' is organized as

$$\mathcal{O}_n^{J,12\prime} = \mathcal{O}_n^{J,12} - \frac{\lambda}{(J+1)\pi^2} \sum_{m=1}^{[J/2]} \delta_{m \neq n} \frac{\sin^2 \frac{\pi n}{J+1} \cos \frac{\pi n}{J+1} \sin^2 \frac{\pi m}{J+1} \cos \frac{\pi m}{J+1}}{\sin^2 \frac{\pi n}{J+1} - \sin^2 \frac{\pi m}{J+1}} \mathcal{O}_m^{J,12}, \tag{3.7}$$

here $[J/2]$ denotes the integer part of $J/2$. This operator redefinition has been considered by us and has been shown not to contribute due to suppression by higher-order powers of $\frac{1}{J}$.

The admixture with fermionic operators is the most difficult to handle. At order λ it is not yet known for the class of symmetric traceless operators considered in this work. The mixing for the trace class operators is derived in eq. (2.1) in [53]. If the mixing for the symmetric traceless sector was described by a formula of the same type as eq. (2.1) in [53] (which is still to be determined whether it is so or not), a rough estimate yields that the mixing might contribute in our case at the order g^2/J^2. However, the tree-level contribution is of order J^2 and the one-loop goes as $g^2 J^0$. Thus the mixing correction will appear at the next $1/J^2$ order while holding λ' order fixed, so that it would not contribute.

Anyway, since we find complete identity between the string and gauge theory calculations this is a clear sign that at the given orders in λ' and $\frac{1}{J}$ no extra mixing has to be taken into account. Whatever the mixing is, presumably it takes place both on the string and field theory sides and gives the same contribution, so that the coincidence of the results might be accounted for. Surely this issue deserves further investigation.

Some comments on the classification and evaluation of these diagrams are necessary. They all arise from expressions with three sums $\sum_{l_1=0,l_2=0,l_3=0}^{J_1,J_2,J}$ over the three wave functions $\psi_{n_1 l_1}, \psi_{n_2 l_2}, \psi_{n_3 l_3}$. The diagrams are done in the planar limits, thus only combinations with
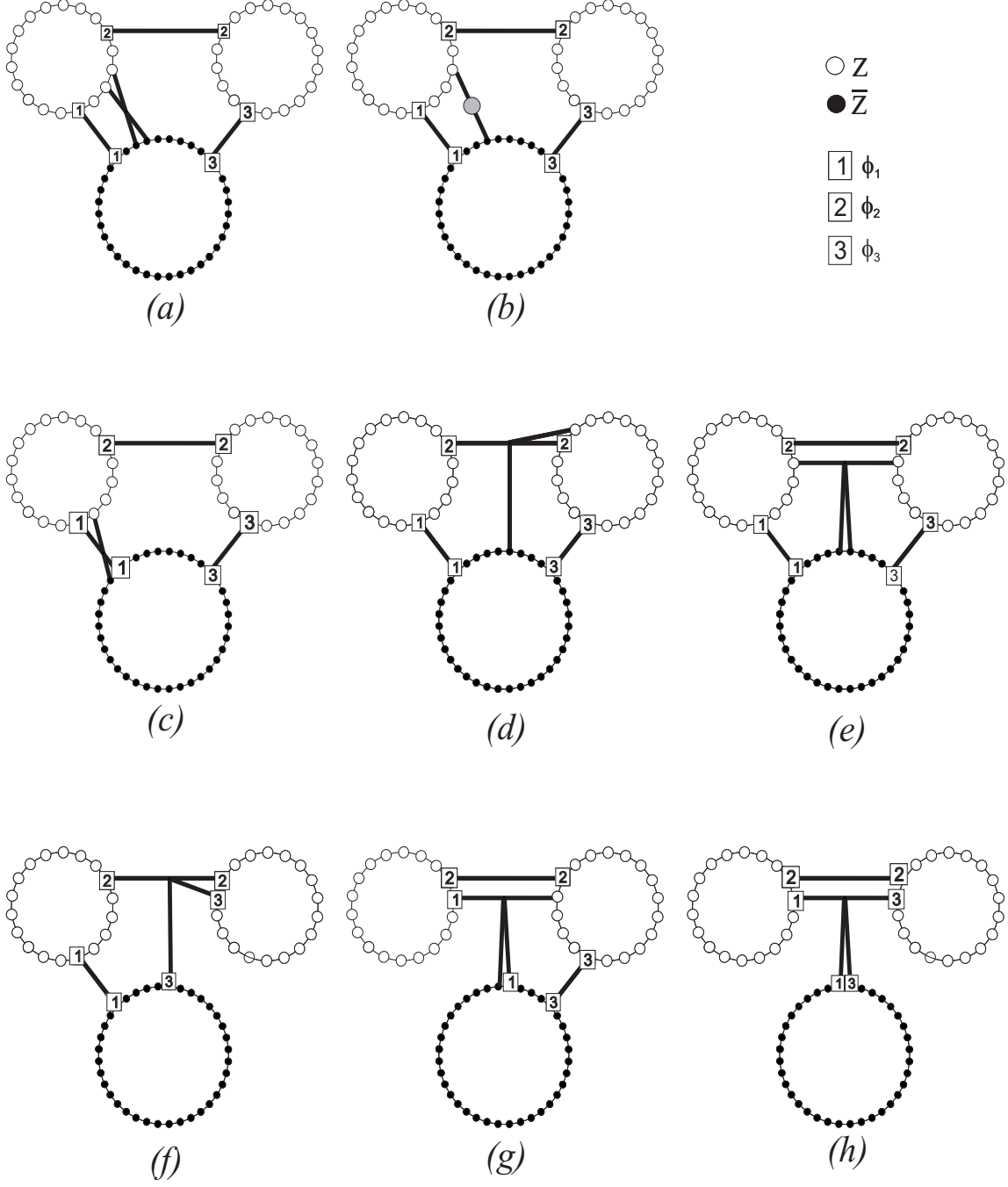
Figure 2. Eight diagrams that contribute to the three-point correlation function of the fully dynamic BMN operators from the symmetric traceless sector C_{123} at one-loop level.

non-intersecting propagators are being considered. One of the sums is always lifted by conservation law. The diagrams (a) and (b) scale as J^3. The diagrams $(c), (d), (e)$ contribute as J^2, $(f), (g)$ scale as J and (h) scales as J^0.

In cases (a) and (b) the self energy of Z or ϕ, or the Z^4 scattering insertion produce an extra factor: l_1 and l_2 fixed, the insertion can be inserted into $\sim J$ locations without breaking the planarity. This raises the order of the diagrams up to J^3; happily their leading order terms do cancel between themselves, and the remaining J^2 and J terms cancel with the rest. The other diagrams do not give rise to extra factors. Also note that while $(c), (d), (e)$ can be realized with arbitrary l_1, l_2, the $(f), (g)$ exist only for marginal cases, with either $l_1 = 0$ or $l_2 = 0$ (we do not show in Fig. (2) the diagrams that differ from $(a) - (h)$ by the $1 \leftrightarrow 2$ symmetry only). We systematize these contributions in the table (1) below. Notice that in principle we could have included explicitly the scheme dependence into the "two-point"-type diagrams $(a) - (c)$ (where the contribution of the operator containing impurities of type 2 and 3 can be factor out) and the "three-point"-type diagrams $(d) - (h)$. The two groups of diagrams are in fact scheme dependent, however the scheme dependence cancels exactly once the two groups are added together. Such cancellation, that leads to the scheme-independence of the full result, is the same as the one discussed in [27] and [28].

Table 1. Classification of diagrams at one-loop level for the three-point correlator of BMN operators

Diagram	Vertex type	Order	Coefficient
a	Z^4	J^3	1
b	Self-energy	J^3	-1
c	$Z^2 \phi^2$	J^2	1
d	$Z^2 \phi^2$	J^2	$1/2$
e	Z^4	J^2	$1/2$
f	ϕ^4	J	1
g	$Z^2 \phi^2$	J	1
h	ϕ^4	1	1

These diagrams are evaluated as

$$S_a^{(1)} = \sum_{l_1=2}^{l_1=J_1} \sum_{l_2=0}^{l_2=J_2} (l_1 - 1) \cos\frac{\pi n_1(2l_1+1)}{J_1+1} \cos\frac{\pi n_2(2l_2+1)}{J_2+1} \cos\frac{\pi n_3(2(l_1+l_2)+1)}{J+1} +$$

$$+ \sum_{l_1=0}^{l_1=J_1-2} \sum_{l_2=0}^{l_2=J_2} (J_1 - l_1 - 1) \cos\frac{\pi n_1(2l_1+1)}{J_1+1} \cos\frac{\pi n_2(2l_2+1)}{J_2+1} \cos\frac{\pi n_3(2(l_1+l_2)+1)}{J+1} + (1 \leftrightarrow 2),$$

$$S_b^{(1)} = (J+3) \sum_{l_1=0}^{l_1=J_1} \sum_{l_2=0}^{l_2=J_2} \cos\frac{\pi n_1(2l_1+1)}{J_1+1} \cos\frac{\pi n_2(2l_2+1)}{J_2+1} \cos\frac{\pi n_3(2(l_1+l_2)+1)}{J+1},$$

$$S_c^{(1)} = \sum_{l_1=0}^{l_1=J_1-1} \sum_{l_2=0}^{l_1=J_2} \cos\frac{\pi n_1(2l_1+1)}{J_1+1} \cos\frac{\pi n_2(2l_2+1)}{J_2+1} \cos\frac{\pi n_3(2(l_1+l_2+1)+1)}{J+1} +$$

$$+ \sum_{l_1=0}^{l_1=J_1} \sum_{l_2=0}^{l_2=J_2-1} \cos\frac{\pi n_1(2l_1+1)}{J_1+1} \cos\frac{\pi n_2(2l_2+1)}{J_2+1} \cos\frac{\pi n_3(2(l_1+l_2+1)+1)}{J+1} +$$

$$+ \sum_{l_1=1}^{l_1=J_1} \sum_{l_2=0}^{l_2=J_2} \cos\frac{\pi n_1(2l_1+1)}{J_1+1} \cos\frac{\pi n_2(2l_2+1)}{J_2+1} \cos\frac{\pi n_3(2(l_1+l_2-1)+1)}{J+1}$$

$$+ \sum_{l_1=0}^{l_1=J_1} \sum_{l_2=1}^{l_1=J_2} \cos\frac{\pi n_1(2l_1+1)}{J_1+1} \cos\frac{\pi n_2(2l_2+1)}{J_2+1} \cos\frac{\pi n_3(2(l_1+l_2-1)+1)}{J+1},$$

$$S_d^{(1)} = S_c,$$

$$S_e^{(1)} = \sum_{l_1=1}^{l_1=J_1} \sum_{l_2=1}^{l_2=J_2} \cos\frac{\pi n_1(2l_1+1)}{J_1+1} \cos\frac{\pi n_2(2l_2+1)}{J_2+1} \cos\frac{\pi n_3(2(l_1+l_2)+1)}{J+1} +$$

$$+ \sum_{l_1=0}^{l_1=J_1-1} \sum_{l_2=0}^{l_2=J_2-1} \cos\frac{\pi n_1(2l_1+1)}{J_1+1} \cos\frac{\pi n_2(2l_2+1)}{J_2+1} \cos\frac{\pi n_3(2(l_1+l_2)+1)}{J+1},$$

$$S_f^{(1)} = \sum_{l_1=0}^{l_1=J_1} \cos\frac{\pi n_1(2l_1+1)}{J_1+1} \cos\frac{\pi n_2}{J_2+1} \cos\frac{\pi n_3(2l_1+1)}{J+1} + (1 \leftrightarrow 2),$$

$$S_g^{(1)} = \sum_{l_1=1}^{l_1=J_1} \cos \frac{\pi n_1 (2l_1 + 1)}{J_1 + 1} \cos \frac{\pi n_2}{J_2 + 1} \cos \frac{\pi n_3 (2(l_1 - 1) + 1)}{J + 1} + (1 \leftrightarrow 2),$$

$$S_h^{(1)} = \cos \frac{\pi n_1}{J_1 + 1} \cos \frac{\pi n_2}{J_2 + 1} \cos \frac{\pi n_3}{J + 1} + (1 \leftrightarrow 2).$$

All these contributions carry also the overall factor $\frac{\lambda}{16\pi^2}$ from the one-loop interaction. They will also carry numerical factors c_i coming from the loop integration. The loop integration and the resulting divergency structure entering diagrams (a) and (e) is fully identical to the one done for the two-point functions in [54]; the diagrams (d) and (e) yield twice as less divergency as (a) or (c). These factors are then found from Table (1) $c_a = 1, c_b = -1, c_c = 1, c_d = 1/2, c_e = 1/2, c_f = 1, c_g = 1, c_h = 1$. The total one loop contribution will be then given by

$$S^{(1)} = \frac{\lambda}{16\pi^2} \sum_i S_i^{(1)} c_i. \tag{3.8}$$

Summing everything up we get

$$S^{(1)} = -\frac{4n^3 \left(n_1^2 (1-y)^2 + n_2^2 y^2 + n_3^2 y^2 (1-y)^2\right) \sin^2(\pi n_3 y)}{\left(n_3^2 y^2 - n_1^2\right) \left(n_3^2 (1-y)^2 - n_2^2\right)} \tag{3.9}$$

whence we get

$$c_{123}^1 = \frac{1}{\lambda'} \frac{S^{(1)}}{S^{(0)}} = -\frac{1}{4} \left(\frac{n_1^2}{y^2} + \frac{n_2^2}{(1-y)^2} + n_3^2\right). \tag{3.10}$$

exactly as in the string theory above.

4   Discussion

We have observed that a three-point correlation function for all dynamical BMN operators matches precisely the perturbative weakly coupled planar field theory and the Penrose limit of the strongly coupled string field theory at one loop level in the Frolov–Tseytlin limit. This result is quite unexpected, since, on one hand, a correlator of two heavy and one light operators has been previously demonstrated in [30] to fail to match the semiclassical string calculation in the Frolov–Tseytlin limit. On the other hand, a heavy-heavy-light correlator calculated via integrability has been shown to beautifully agree with the string theory in the Frolov–Tseytlin limit, yet only in the thermodynamical regime, when the number of excitations tends to infinity [23]. Our result is thus the only one-loop analytic calculation of a three-point function so far, where complete agreement between fields and strings is observed. It has been noted in [30] that such a matching is not necessarily present even at one-loop level, since C_{123} is unprotected. Thus our case should be considered as another "wonder" of AdS/CFT and must be explained somehow. The well known state/operator identification for BMN states, which in other cases is not so well established [30, 31], certainly helps in providing this matching. However, we do not yet possess a generic argument why this must work in a more general setting; neither we can guess which corner of the parameter space may

be covered by the conjecture on exact matching between the three-point functions on gauge and gravity sides. The Penrose limit string field theory Hamiltonian which is the basis of our string calculation seems to know nothing about the Yang-Mills planar correspondence, yet it reproduces its results astonishingly. On the other hand, Yang-Mills planar theory does indeed produce the terms of order λJ^2, which were supposed by authors of [30] to be exactly the stumbling block in the heavy-heavy-light matching. In the three-BMN case these stumbling blocks cancel each other accurately, leaving only terms of similar orders both in λ and in J on both sides of the correspondence.

Acknowledgments

References

[1] E. Buchbinder and A. Tseytlin, "On semiclassical approximation for correlators of closed string vertex operators in AdS/CFT," *JHEP* **1008** (2010) 057, `arXiv:1005.4516 [hep-th]`.

[2] M. S. Costa, R. Monteiro, J. E. Santos, and D. Zoakos, "On three-point correlation functions in the gauge/gravity duality," *JHEP* **1011** (2010) 141, `arXiv:1008.1070 [hep-th]`.

[3] R. Roiban and A. Tseytlin, "On semiclassical computation of 3-point functions of closed string vertex operators in $AdS_5 x S^5$," *Phys.Rev.* **D82** (2010) 106011, `arXiv:1008.4921 [hep-th]`.

[4] K. Zarembo, "Holographic three-point functions of semiclassical states," *JHEP* **1009** (2010) 030, `arXiv:1008.1059 [hep-th]`.

[5] E. Buchbinder and A. Tseytlin, "Semiclassical correlators of three states with large S^5 charges in string theory in $AdS_5 x S^5$," `arXiv:1110.5621 [hep-th]`.

[6] M. Michalcik, R. C. Rashkov, and M. Schimpf, "On semiclassical calculation of three-point functions in $AdS_5 \times T^{(}1, 1)$," `arXiv:1107.5795 [hep-th]`.

[7] T. Klose and T. McLoughlin, "A light-cone approach to three-point functions in $AdS_5 x S^5$," `arXiv:1106.0495 [hep-th]`.

[8] G. Georgiou, "Two and three-point correlators of operators dual to folded string solutions at strong coupling," *JHEP* **1102** (2011) 046, `arXiv:1011.5181 [hep-th]`.

[9] R. Hernandez, "Three-point correlation functions from semiclassical circular strings," *J.Phys.A* **A44** (2011) 085403, `arXiv:1011.0408 [hep-th]`.

[10] D. Arnaudov and R. Rashkov, "On semiclassical calculation of three-point functions in $AdS_4 \times CP^3$," *Phys.Rev.* **D83** (2011) 066011, `arXiv:1011.4669 [hep-th]`.

[11] C. Park and B.-H. Lee, "Correlation functions of magnon and spike," *Phys.Rev.* **D83** (2011) 126004, `arXiv:1012.3293 [hep-th]`.

[12] J. Russo and A. Tseytlin, "Large spin expansion of semiclassical 3-point correlators in $AdS_5 x S^5$," *JHEP* **1102** (2011) 029, arXiv:1012.2760 [hep-th].

[13] D. Bak, B. Chen, and J.-B. Wu, "Holographic Correlation Functions for Open Strings and Branes," *JHEP* **1106** (2011) 014, arXiv:1103.2024 [hep-th].

[14] D. Arnaudov, R. Rashkov, and T. Vetsov, "Three and four-point correlators of operators dual to folded string solutions in $AdS_5 x S^5$," *Int.J.Mod.Phys.* **A26** (2011) 3403–3420, arXiv:1103.6145 [hep-th].

[15] R. Hernandez, "Three-point correlators for giant magnons," *JHEP* **1105** (2011) 123, arXiv:1104.1160 [hep-th].

[16] X. Bai, B.-H. Lee, and C. Park, "Correlation function of dyonic strings," *Phys.Rev.* **D84** (2011) 026009, arXiv:1104.1896 [hep-th].

[17] C. Ahn and P. Bozhilov, "Three-point Correlation functions of Giant magnons with finite size," *Phys.Lett.* **B702** (2011) 286–290, arXiv:1105.3084 [hep-th].

[18] D. Arnaudov and R. Rashkov, "Quadratic corrections to three-point functions," arXiv:1106.0859 [hep-th].

[19] R. A. Janik and A. Wereszczynski, "Correlation functions of three heavy operators: The AdS contribution," arXiv:1109.6262 [hep-th].

[20] J. Escobedo, N. Gromov, A. Sever, and P. Vieira, "Tailoring Three-Point Functions and Integrability II. Weak/strong coupling match," *JHEP* **1109** (2011) 029, arXiv:1104.5501 [hep-th].

[21] J. Escobedo, N. Gromov, A. Sever, and P. Vieira, "Tailoring Three-Point Functions and Integrability," *JHEP* **1109** (2011) 028, arXiv:1012.2475 [hep-th].

[22] N. Gromov, A. Sever, and P. Vieira, "Tailoring Three-Point Functions and Integrability III. Classical Tunneling," arXiv:1111.2349 [hep-th].

[23] N. Gromov and P. Vieira, "Quantum Integrability for Three-Point Functions," arXiv:1202.4103 [hep-th]. 4 pages, 3 figures.

[24] D. Serban, "A note on the eigenvectors of long-range spin chains and their scalar products," arXiv:1203.5842 [hep-th].

[25] I. Kostov, "Classical Limit of the Three-Point Function from Integrability," arXiv:1203.6180 [hep-th].

[26] G. Grignani and A. Zayakin, "Matching Three-point Functions of BMN Operators at Weak and Strong coupling," *JHEP* **06** 142.

[27] K. Okuyama and L.-S. Tseng, "Three-point functions in N = 4 SYM theory at one-loop," *JHEP* **0408** (2004) 055, arXiv:hep-th/0404190 [hep-th].

[28] L. F. Alday, J. R. David, E. Gava, and K. Narain, "Structure constants of planar N = 4 Yang Mills at one loop," *JHEP* **0509** (2005) 070, arXiv:hep-th/0502186 [hep-th].

[29] G. Georgiou, V. Gili, A. Grossardt, and J. Plefka, "Three-point functions in planar N=4 super Yang-Mills Theory for scalar operators up to length five at the one-loop order," arXiv:1201.0992 [hep-th]. 45, 5 tables, many figures. arXiv admin note: substantial text overlap with arXiv:1007.2356.

[30] A. Bissi, T. Harmark, and M. Orselli, "Holographic 3-point function at one loop," arXiv:1112.5075 [hep-th].

[31] A. Bissi, C. Kristjansen, D. Young, and K. Zoubos, "Holographic three-point functions of giant gravitons," *JHEP* **1106** (2011) 085, arXiv:1103.4079 [hep-th].

[32] O. Foda, "N=4 SYM structure constants as determinants," arXiv:1111.4663 [math-ph]. 26 pages. Added more introductory material and more references, improved the presentation and corrected (a large number of) typos, particularly the references to the figures. Result is unchanged.

[33] S. Frolov and A. A. Tseytlin, "Rotating string solutions: AdS / CFT duality in nonsupersymmetric sectors," *Phys.Lett.* **B570** (2003) 96–104, arXiv:hep-th/0306143 [hep-th].

[34] T. Harmark, K. R. Kristjansson, and M. Orselli, "Matching gauge theory and string theory in a decoupling limit of AdS/CFT," *JHEP* **0902** (2009) 027, arXiv:0806.3370 [hep-th].

[35] D. E. Berenstein, J. M. Maldacena, and H. S. Nastase, "Strings in flat space and pp waves from N=4 superYang-Mills," *JHEP* **0204** (2002) 013, arXiv:hep-th/0202021 [hep-th].

[36] M. Spradlin and A. Volovich, "Superstring interactions in a pp wave background. 2.," *JHEP* **0301** (2003) 036, arXiv:hep-th/0206073 [hep-th].

[37] S. Dobashi, H. Shimada, and T. Yoneya, "Holographic reformulation of string theory on AdS(5) x S**5 background in the PP wave limit," *Nucl.Phys.* **B665** (2003) 94–128, arXiv:hep-th/0209251 [hep-th]. To the memory of Prof. Bunji Sakita.

[38] J. Pearson, M. Spradlin, D. Vaman, H. L. Verlinde, and A. Volovich, "Tracing the string: BMN correspondence at finite J**2/N," *JHEP* **0305** (2003) 022, arXiv:hep-th/0210102 [hep-th].

[39] A. Pankiewicz and J. Stefanski, B., "PP wave light cone superstring field theory," *Nucl.Phys.* **B657** (2003) 79–106, arXiv:hep-th/0210246 [hep-th].

[40] Y.-H. He, J. H. Schwarz, M. Spradlin, and A. Volovich, "Explicit formulas for Neumann coefficients in the plane wave geometry," *Phys.Rev.* **D67** (2003) 086005, arXiv:hep-th/0211198 [hep-th].

[41] A. Pankiewicz and J. Stefanski, B., "On the uniqueness of plane wave string field theory," arXiv:hep-th/0308062 [hep-th].

[42] P. Di Vecchia, J. L. Petersen, M. Petrini, R. Russo, and A. Tanzini, "The Three string vertex and the AdS / CFT duality in the PP wave limit," *Class.Quant.Grav.* **21** (2004) 2221–2240, arXiv:hep-th/0304025 [hep-th].

[43] S. Dobashi and T. Yoneya, "Resolving the holography in the plane-wave limit of AdS/CFT correspondence," *Nucl.Phys.* **B711** (2005) 3–53, arXiv:hep-th/0406225 [hep-th].

[44] S. Dobashi and T. Yoneya, "Impurity non-preserving 3-point correlators of BMN operators from PP-wave holography. I. Bosonic excitations," *Nucl.Phys.* **B711** (2005) 54–82, arXiv:hep-th/0409058 [hep-th].

[45] H. Shimada, "Holography at string field theory level: Conformal three point functions of BMN operators," *Phys.Lett.* **B647** (2007) 211–218, arXiv:hep-th/0410049 [hep-th].

[46] S. Lee and R. Russo, "Holographic cubic vertex in the pp-wave," *Nucl.Phys.* **B705** (2005) 296–318, `arXiv:hep-th/0409261 [hep-th]`.

[47] G. Grignani, M. Orselli, B. Ramadanovic, G. W. Semenoff, and D. Young, "Divergence cancellation and loop corrections in string field theory on a plane wave background," *JHEP* **0512** (2005) 017, `arXiv:hep-th/0508126 [hep-th]`.

[48] G. Grignani, M. Orselli, B. Ramadanovic, G. W. Semenoff, and D. Young, "AdS/CFT versus string loops," *JHEP* **0606** (2006) 040, `arXiv:hep-th/0605080 [hep-th]`.

[49] C. Kristjansen, J. Plefka, G. Semenoff, and M. Staudacher, "A New double scaling limit of N=4 superYang-Mills theory and PP wave strings," *Nucl.Phys.* **B643** (2002) 3–30, `arXiv:hep-th/0205033 [hep-th]`.

[50] N. Beisert, C. Kristjansen, J. Plefka, G. Semenoff, and M. Staudacher, "BMN correlators and operator mixing in N=4 superYang-Mills theory," *Nucl.Phys.* **B650** (2003) 125–161, `arXiv:hep-th/0208178 [hep-th]`.

[51] N. Beisert, C. Kristjansen, and M. Staudacher, "The Dilatation operator of conformal N=4 superYang-Mills theory," *Nucl.Phys.* **B664** (2003) 131–184, `arXiv:hep-th/0303060 [hep-th]`.

[52] G. Georgiou, V. L. Gili, and R. Russo, "Operator Mixing and the AdS/CFT correspondence," *JHEP* **0901** (2009) 082, `arXiv:0810.0499 [hep-th]`.

[53] G. Georgiou, V. L. Gili, and R. Russo, "Operator mixing and three-point functions in N=4 SYM," *JHEP* **0910** (2009) 009, `arXiv:0907.1567 [hep-th]`.

[54] J. Minahan and K. Zarembo, "The Bethe ansatz for N=4 superYang-Mills," *JHEP* **0303** (2003) 013, `arXiv:hep-th/0212208 [hep-th]`.